Greiter and Schuricht reply: In response to a recent manuscript of ours [1], Bernevig, Giuliano, and Laughlin (BGL) have reiterated their claim that spinons in the Haldane–Shastry model (HSM) are interacting in a comment posted on cond-mat [2]. Notwithstanding that this comment does not contain any truly new arguments, that is, no argument which has not been presented in some form or variation in one of their previous papers [3,4] and subsequently been disputed in our manuscript [1], we feel a certain obligation to respond to each point they make. We should caution, however, that some readers may perceive the discussion as circular.

To begin with, BGL acknowledge that the S-matrix for spinon-spinon scattering in the HSM does not depend on the pseudomomenta used in the framework of the asymptotic Bethe Ansatz (ABA) [5], while maintaining that there is an spinon-spinon interaction. They claim that "the apparent contradiction [...] is just a consequence of a different way of labeling spinons". In other words, they assert that the spinon S-matrix does depend on the true and physical spinon momenta, while it does not depend on the pseudomomenta (which they refer to as "quasimomenta"), which, according to them, "are good quantum numbers when exactly solving correlated Hamiltonians but [...] are not observable".

It is not very difficult to see that this line of thought is unsustainable. The pseudomomenta in the ABA solutions are the quantum numbers which label the exact eigenstates of the Hamiltonian, and in particular the spin polarized two-spinon eigenstates investigated by BGL. Regardless of the physical interpretation of these numbers, if the spinon scattering matrix S does not depend on them, it does not depend on the quantum numbers labeling the states. Hence S cannot depend on the true and physical spinon momenta either, whatever they may be. The scattering matrix is an unambiguously defined quantity. In the case of spinons in the HSM, $S = i$ directly and unambivalently implies that the spinons are non-interacting particles with half-fermi statistics.

BGL further emphasize that the decomposition of the basis states $\Psi_{\alpha\beta}$ in terms of the energy eigenstates Φ_{mn} is unambiguous, and that the latter basis does not suffer from overcompleteness. This is in no way contrary to our observations: it is the basis $\Psi_{\alpha\beta}$ which is overcomplete, and this renders an interpretation of α and β as spinon coordinates ambiguous. It is hence neither possible to interpret $p_{nm}(\eta_\alpha - \eta_\beta)$ as the relative wave function of the two spinons, nor possible to interpret an enhancement in this quantity for short separations as evidence for an attraction, as explained at length in [1]. Haldane had a good reason to refer to α and β as "spinon coordinates" in quotation marks [6].

Finally, BGL claim that the identification of the individual spinon momenta we propose ((17) of [1]) is unphysical. In this context, we wish to remark that it is only possible to read of the total momentum of the state (7) (the equation numbers here and below refer to [1]) that is, the sum of the momenta of the two spinons,

$$q_m + q_n = \pi - \frac{2\pi}{N}\,(m + n + 1)\,,$$

where $m \geq n$. BGL assign the individual spinon momenta according to (16), while we propose (17). Let us emphasize here that there is no reason to assume that the functional form of both q_m and q_n as functions of m and n (as $m \geq n$) should be the same. Our proposal (17) has no drawbacks in comparison (16), but the advantage that the relative momentum spacing between the spinons take the values appropriate for half-fermions, while the spacings of (16) would be appropriate for bosons. (It was established by Haldane that the spinons are half-fermions [7].) Our expression for the two-spinon energies (18) is consistent with the Bethe Ansatz result, while the corresponding expression obtained with BGL's proposal (16) would yield a spinon-spinon interaction term. The ABA solution of the model, however, precludes such an interaction, as reemphasized above. In our opinion, it is fair to conclude that (17) is the physically correct assignment. In any occasion, it is not in the scientific tradition to base a claim on rather arbitrarily assuming (16).

In the conclusion of their comment, BGL state that the ABA result, which states that the spinons in the HSM are free, does not contradict their conclusion that they are interacting, but rather complements it. In fact, however, these statements do contradict each other, and there are only two possible resolutions. Either the ABA is not applicable to the model, and therefore not able to provide reliable results, or it is applicable, and the spinons are free. We believe we have unambiguously dispersed all of BGL's evidence against the latter. There is no attraction between spinons in the HSM.

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[1] M. Greiter and D. Schuricht, cond-mat/0409495.
[2] B. A. Bernevig *et al.*, cond-mat/0410147.
[3] B. A. Bernevig *et al.*, Phys. Rev. Lett. **86**, 3392 (2001).
[4] B. A. Bernevig *et al.*, Phys. Rev. B **64**, 24425 (2001).
[5] F. H. L. Eßler, Phys. Rev. B **51**, 13357 (1995).
[6] F. D. M. Haldane, Phys. Rev. Lett. **66**, 1529 (1991).
[7] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).